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Study of neutron reflection from a curved surface using the Monte Carlo method

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STUDY OF NEUTRON REFLECTION FROM A CURVED SURFACE USING THE MONTE CARLO METHOD

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A

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ABSTRACT

The reflection patterns for neutrons impinging on both an infinite parabola and an infinite slab were studied using the Monte Carlo method.

In the first case neutrons were sent into an infinite parabola ($z = x^2/2$) moving on the XZ plane parallel to the z axis at 5 points, namely: $y = 1.5$, $x = -1.5$; $y = 1.5$, $x = -.75$; $y = 1.5$, $x = 0.0$; $y = 1.5$, $x = .75$; $y = 1.5$, $x = 1.5$. The results were that there appeared to be a "focusing" of the scattered neutrons.

In the second case, neutrons were allowed to impinge close to the apex of the parabola at $y = 1.5$, $x = -0.5$; $y = 1.5$, $x = -0.25$; $y = 1.5$, $x = 0.0$; $y = 1.5$, $x = 0.25$; $y = 1.5$, $x = 0.5$. These results were compared to an infinite slab by sending neutrons into the slab at the same points. The parabola pattern showed a definite greater concentration than did the slab pattern.

The above results lead the author to believe that a parabolic reflector might be beneficial in reactor experiments where neutron high density is desired at some point in space.

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I. INTRODUCTION

The purpose of this work is to study the reflection of thermal neutrons from curved surfaces. In particular, the author investigated the possibility of neutrons being "focused" after being reflected from a curved surface (e.g. a parabolic surface) in much the same way as electromagnetic waves can focus.

This idea originated while the author was still in the U.S. Navy. He was interested then in the giant parabolic antennas used in the radar systems. Incoming radio waves have the tendency to reflect back through the focal point of the parabolic antenna. A receiver placed at the focal point of the parabola accepts an intensified incoming signal. If the corresponding phenomenon is true for neutrons, possible applications might be that in a reactor core the fuel would be concentrated at the focal point of a parabolic reflector.

Fermi in 1936 [1] suggested that a neutron which suffers a collision close to the boundary of a medium will have a greater probability of escape if its direction after the collision is along the outward normal to the surface. Consequently, the angular distribution of the escaping neutrons should be peaked in this direction. For the simplified case of thermal neutrons diffusing in a non-capturing and isotropic scattering medium, the emergent angular distribution from a plane surface is given by Fermi's [2] approximate formula:

$$F(\theta) = \frac{\cos\theta + \sqrt{3}\cos^2\theta}{\pi\left[1 + \frac{2}{\sqrt{3}}\right]} \qquad (1)$$

where

θ = the angle between the direction of neutron emission and the normal to the surface.

$F(\theta)$ = number of neutrons emerging per unit area of surface per unit solid angle at angle θ, i.e. $F(\theta)$ represents the angular neutron distribution. As given in Eq. (1), it is normalized to unit current leaving the plane per unit area. The function is shown in Fig. 1. The experimental results were obtained by Hoffman and Livingston [2].

Placzek in 1947 [3] using transport theory made an exact calculation of $F(\theta)$. The agreement with Fermi's Formula is so good that the two distributions can not be distinguished if drawn on the same figure. Cambiaghi et al in 1968 [8] studied neutron focusing by a conical tube experimentally and Shimooke in 1969 [4] did the same thing theoretically.

Generally neutron "focusing" is of interest either in studies of the material which reflects the neutrons or in achieving high neutron densities at certain points in space. The most important application would be in decreasing neutron leakage in a reactor by using parabolic reflectors.

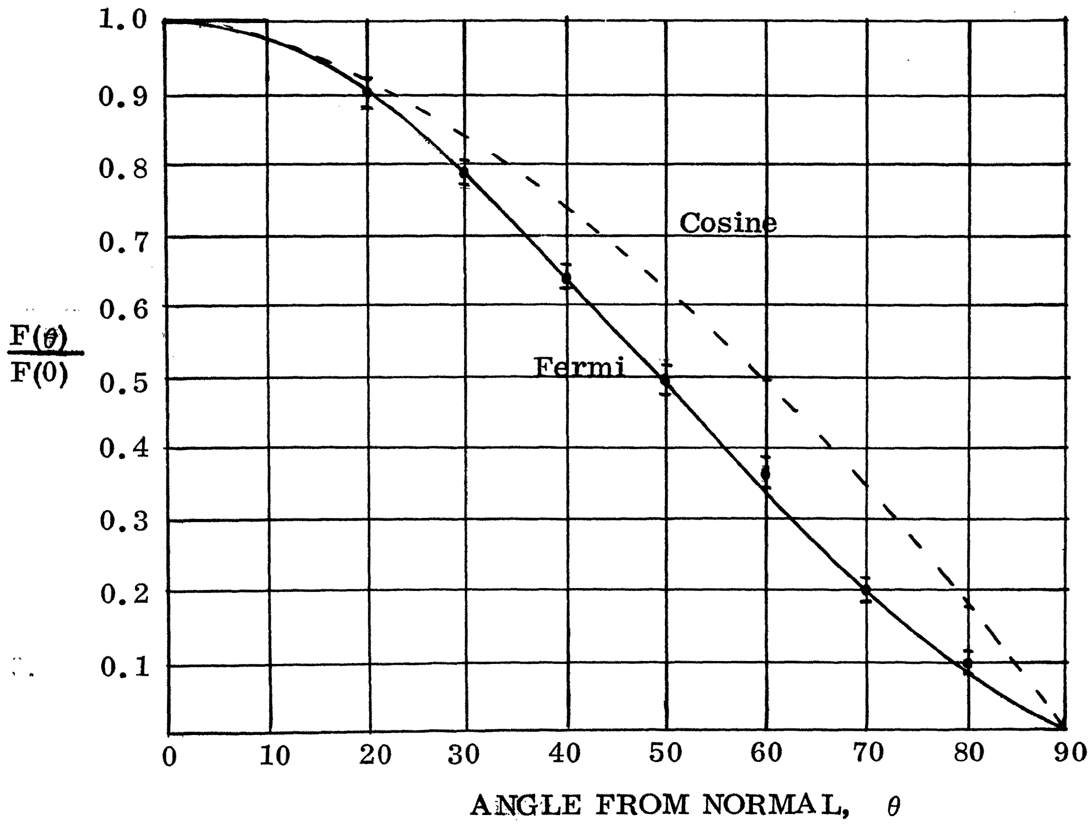


Fig. 1. The Angular Distribution of the Neutron Current Emerging from the Plane Surface of a Half Space [2].

II. DESCRIPTION OF THE PROBLEM

Neutrons impinging on an infinite medium are either:

(1) lost in the medium

(2) reflected.

The reflected neutrons are the ones of interest in this problem, especially when they are reflected from a parabolic reflector. As is well known, plane electromagnetic waves will focus (Fig. 2) when reflected from a parabolic reflector.

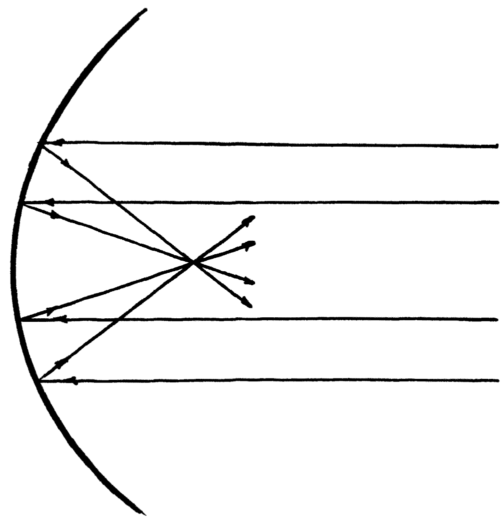


Fig. 2. Electromagnetic Waves Focus When They Impinge upon a Parabolic Reflector.

To study whether or not the reflected neutrons might "focus" using a parabolic reflector one would need to know the neutron flux or current as a function of r and Ω. The exact solution would be that of the Boltzmann equation, but this is very difficult if not impossible. Even if the diffusion equation is used, which is an approximation to the Boltzmann equation, the problem is still a difficult one.

The steady state diffusion equation is as follows:

$$D(E)\nabla^2 \varphi(E,\vec{r}) - \Sigma_a(E)\,\varphi(E,\vec{r}) + S(E,\vec{r}) = 0 \tag{2}$$

where

$n(E, r)$ = neutron density at energy E, at point $\vec{r}$

$D(E)$ = diffusion coefficient at energy, E

$\varphi(E, \vec{r})$ = neutron flux at energy E, at point $\vec{r}$

$\Sigma_a(E)$ = absorption cross section at energy E

$S(E, \vec{r})$ = source of neutrons of energy E, at point $\vec{r}$

Assuming

(1) Thermal neutrons, i.e., energy dependence is suppressed. (Quantities like D and Σ_a are averages over the thermal neutron spectrum.)

(2) Non-multiplying medium (Source term is zero.)

the equation becomes:

$$D\nabla^2 \varphi - \Sigma_a \varphi = 0 \tag{3}$$

or

$$\nabla^2 \varphi - \frac{1}{L^2}\varphi = 0 \tag{4}$$

where

$$\frac{\Sigma_a}{D} = \frac{1}{L^2} \tag{5}$$

Since the system of interest is parabolic, we introduce the Laplacian operator for parabolic coordinates, [5] namely

$$\nabla^2 = \frac{1}{u^2 - v^2}\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2}\right) + \frac{\partial^2}{\partial z^2} \tag{6}$$

where

$$x = \frac{1}{2}(u^2 - v^2), \quad y = uv, \quad z = z \tag{7}$$

If v = c, the Laplacian reduces to

$$\nabla^2 = \frac{1}{2x} \left(\frac{\partial^2}{\partial u^2} \right) + \frac{\partial^2}{\partial z^2} \tag{8}$$

Substituting Eq. (15) into Eq. (11) we get

$$\frac{1}{2x} \frac{\partial^2 \varphi}{\partial u^2} + \frac{\partial^2 \varphi}{\partial z^2} = \frac{1}{L^2} \varphi \tag{9}$$

The $\frac{1}{2x}$ term makes the solution of this partial differential equation difficult to express in closed form. The equation can be solved by the use of a computer, but Eq. (9) is not valid at the boundaries. Since solution at the boundaries is important in the present problem, instead of working with Eq. (9), the Monte Carlo Technique was used to obtain the desired results.

The Monte Carlo Technique gets its name from the fact that in all the various forms in which it is applied, a random sampling process is involved. The Monte Carlo Technique studies the individual particles, and after the history of a sufficient number of particles is followed, conclusions can be drawn as to the average behavior of the particles. This is in contrast to the solution of differential equations where the average neutron is studied. In the computer program written, decisions are made as to the fate of the neutron through the use of random numbers and probability functions related to nuclear behavior, and consistent with the known probabilities for individual interactions.

III. COMPUTER PROGRAM

In the program the following assumptions are made.

1. Infinite Medium

Since this study is concerned with only the neutrons reflected, without effects of boundaries, a finite system would not yield additional information.

2. Isotropic Scattering

In thermalization problems the scattering cross section is taken as isotropic in the laboratory system. This is equivalent to keeping only the first term in a Legendre polynomial expansion of the scattering cross section. The higher terms of the Legendre expansion are proportional to powers of the ratio of the neutron mass to the scattering atomic mass. In the thermal region the effective scattering mass is several times higher than the free atomic mass, due to chemical binding effects. Thus, the higher terms should be small [9].

3. 150 Collisions

To reduce computer time, a neutron was considered lost if still in the medium after 150 collisions.

A description of the coordinate system used in the computer program is included in Appendix A.

A copy of the computer program itself is included in Appendix B.

A flow diagram is shown in Figs. 3 and 4.

Comments:

(1) The probabilities mentioned in statement (1) are

$$PA = \Sigma_a/\Sigma_t, \quad PS = \Sigma_s/\Sigma_t \qquad (10)$$

where

PA = probability the neutron will be absorbed in an interaction,

PS = probability the neutron will be scattered in an interaction,

Σ_a = absorption cross section,

Σ_s = scatter cross section,

$\Sigma_t = \Sigma_a + \Sigma_s$

(2) Calculation of the distance between two successive interactions (statement 4) was done as follows:

$e^{-\Sigma_t x}$ = probability that a neutron will travel a distance x without an interaction.

$e^{-\Sigma_t x}\Sigma_t dx$ = probability there will be an interaction between x and x + dx.

$P(x) = \int_0^x e^{-\Sigma_t x}\Sigma_t dx$ = probability of an interaction occurring between x = 0 and x = x. $(0 \leq P(\infty) \leq 1)$

P(x) is determined by the help of a random number S. The random number S $(0 \leq S \leq 1)$ is set equal to P(x). Then

$$S = \int_0^x e^{-\Sigma_t x}\Sigma_t dx$$

$$= 1 - e^{-\Sigma_t x}$$

$$e^{-\Sigma_t x} = 1 - S$$

$$-\Sigma_t x = \ln(1 - S)$$

$$x = -\frac{1}{\Sigma_t}\ln(1 - S) \qquad (11)$$

If S is a random number, 1 - S would also be a random number. Hence, an equation that is just as valid is

$$x = -\frac{1}{\Sigma_t} \ln(S) \tag{12}$$

Lambda is the symbol for x in the program.

(3) In statement (8) the cartesian coordinates are calculated after each interaction (See Appendix A).

(4) Calculation of the point where a neutron trajectory intersects a plane of interest is as follows:

Suppose the plane of interest is a Z plane.(See Fig. 5). The program from statement (8) has the value of x, y, z, θ, and φ at the point after the last collision, and for the parabola (See Appendix B).

$$DZP = (Z - Zp) / \cos\varphi \sin\theta \tag{13}$$

where

DZP = vector distance to the Z plane from the point (x, y, z). Everything on the right side of Eq. (13) is known, so x and y of the point of penetration is calculated from:

$$x_{penetration} = x_{(from\ (8))} + DZP * \cos\varphi * \cos\theta \tag{14}$$

$$y_{penetration} = y_{(from\ (8))} + DZP * \sin\varphi \tag{15}$$

The rest of the flow diagram is fairly self explanatory.

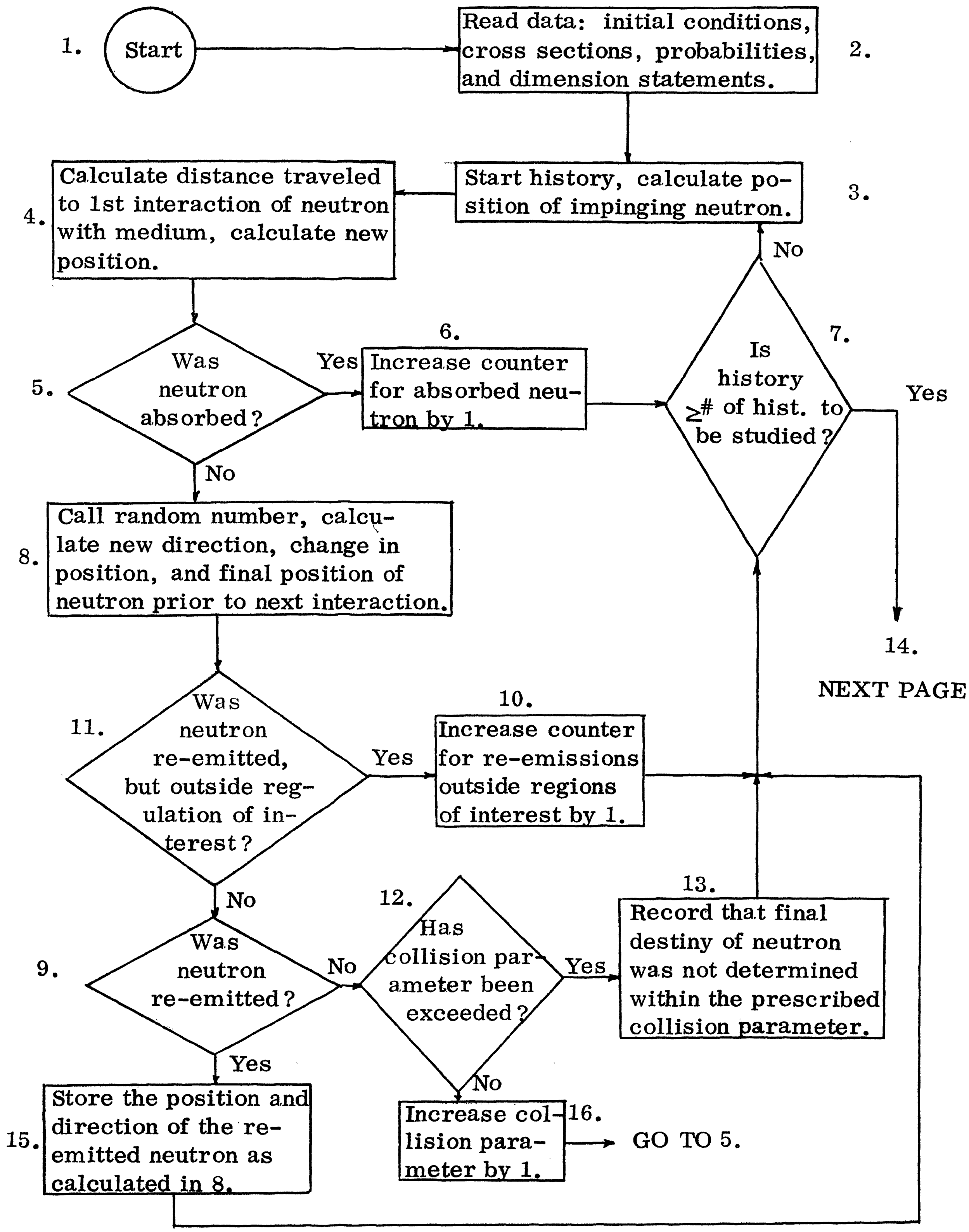


Fig. 3. Flow Diagram.

At this point in the program all the histories have been studied and the remaining program calculates where the re-emitted neutrons penetrate the planes of interest.

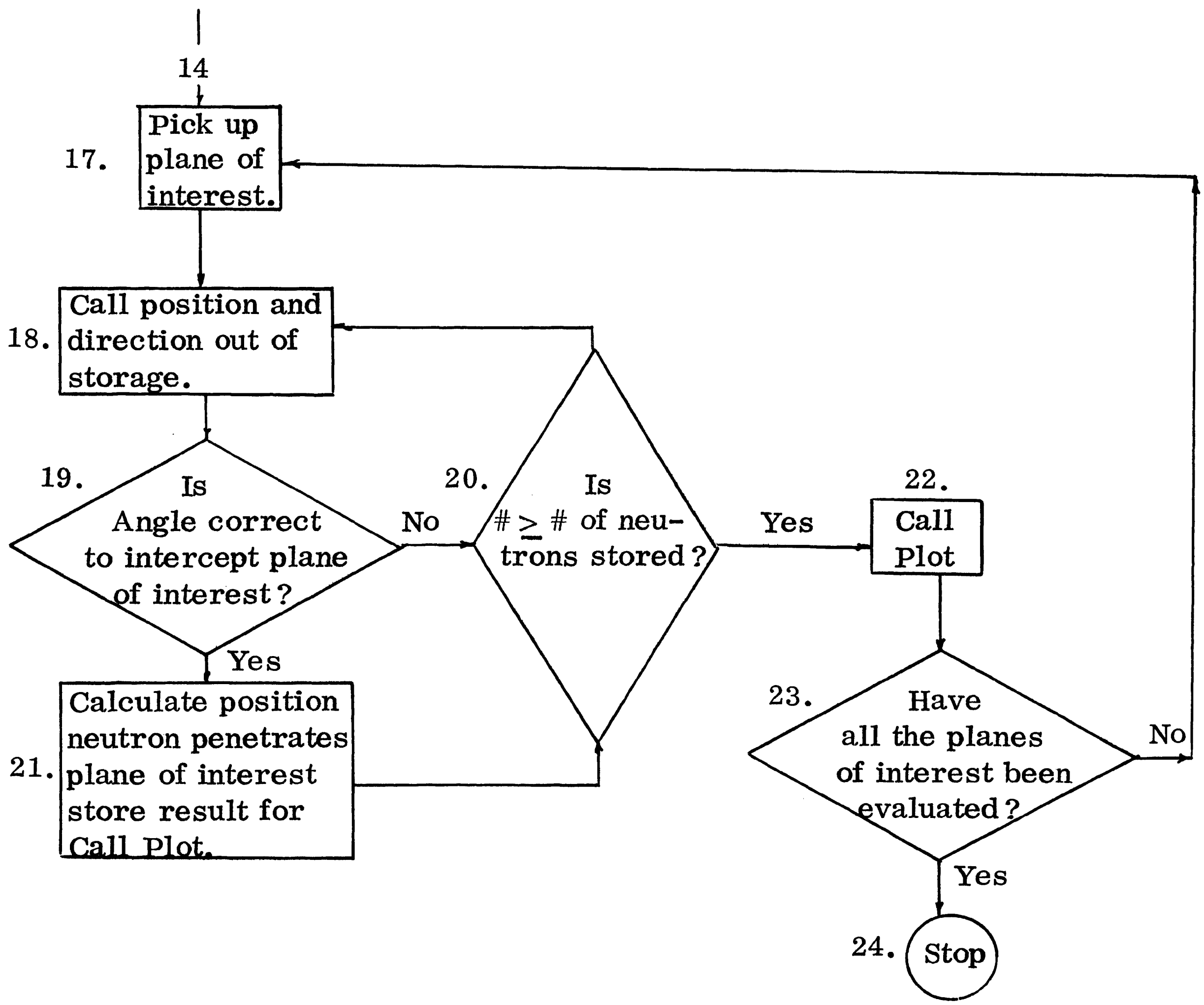


Fig. 4. Continuation of Flow Diagram.

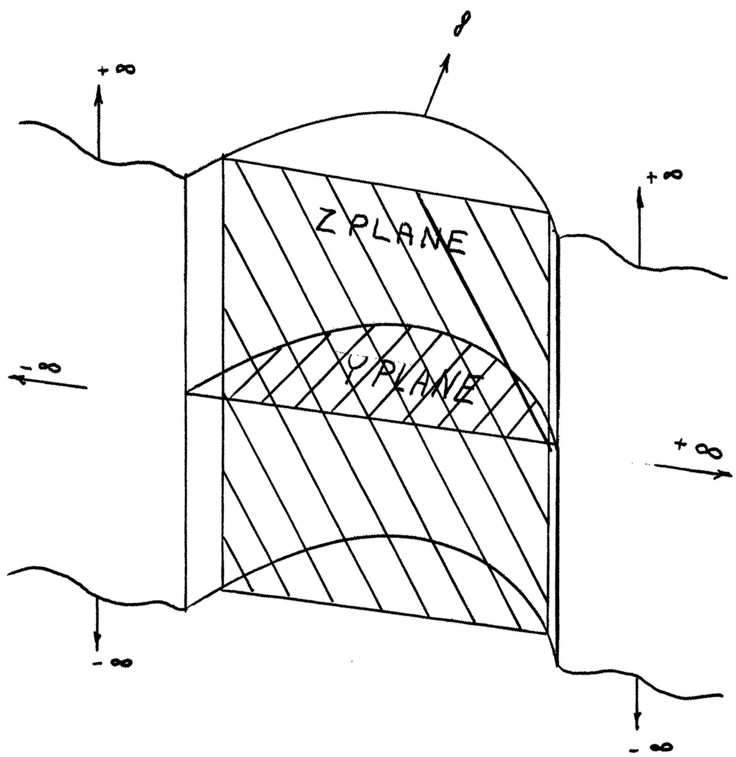


Fig. 5. Isometric Sketch Showing Z and Y Plane Orientation Used in Computer Program.

IV. RESULTS

In one of the computer programs 300 neutrons are sent in (one at a time) at each of the following positions: $y = 1.5$, $x = -1.5$; $y = 1.5$, $x = -.75$; $y = 1.5$, $x = 0.0$; $y = 1.5$, $x = .75$; $y = 1.5$, $x = 1.5$.

All incoming neutrons are traveling on the Y plane (Fig. 5) parallel to the z axis. Thus, the combined effect of all incoming neutrons corresponds to a parallel neutron beam striking the reflector at $y = 1.5$. The program then selects Y planes at $y = 0.75$, 1.50, 2.25, and Z planes at $z = 0.0$ to 1.0 for the infinite parabola. When a plane is selected, the distance to the plane from the point of the reflected neutron is calculated. Then, the coordinates of where the neutron penetrates the selected plane are stored in the computer memory. When all points of penetration are stored, a plotting subroutine is used to plot out the stored points on a graph.

Two other programs were run where 300 neutrons were sent in at the following positions: $y = 1.5$, $x = -0.5$; $y = 1.5$, $x = -0.25$; $y = 1.5$, $x = 0.0$; $y = 1.5$, $x = 0.25$; and at $y = 1.5$, $x = 0.5$. One program was for an infinite parabola, the other for an infinite slab. The purpose of these last two programs was to see if the shape of the parabolic reflector would have any effect on the neutrons when they impinge on a surface shape approaching that of the plane wall. The points where the neutrons entered the medium in these two programs are so close to the apex of the parabola that the incoming neutron beam "sees" a flat surface.

The medium considered in the present work was graphite. However, since isotropic scattering was used and crystalline effects were neglected, the results are valid to within a constant for any other medium for which the same approximations can be used. The differences in the results between two media will come from the differences in the values of scattering and absorption cross sections.

The results of this project are presented as a series of graphs and tables.

Figs. 6-12 are the results of the first computer program mentioned. Figs. 6, 7, and 8 show how the Y planes y = 1.00, y = 1.50, and y = 2.00 for the infinite parabola were penetrated by the re-emerging neutrons. Each * represents a point of penetration. See Appendix A for description of geometry used and Fig. 5 for the orientation of the planes. The explanation for the points that lie outside the curved boundary is that no allowance was made for secondary interactions in the computer program, so the points that look illogical would have had a collision with the curved surface before penetrating the plane. The amount of penetrations of this type is a relatively small number when compared to the total number of penetrations (less than 3%).

Figs. 9, 10, 11, and 12 show how the Z planes z = 0.0, 0.25, 0.5, and 0.75 for the infinite parabola were penetrated by the re-emerging neutrons.

Figs. 13-16 are the results of the second set of programs mentioned. They show results for Y plane = 1.5, and for the Z planes 0.250, 0.375, and 0.500.

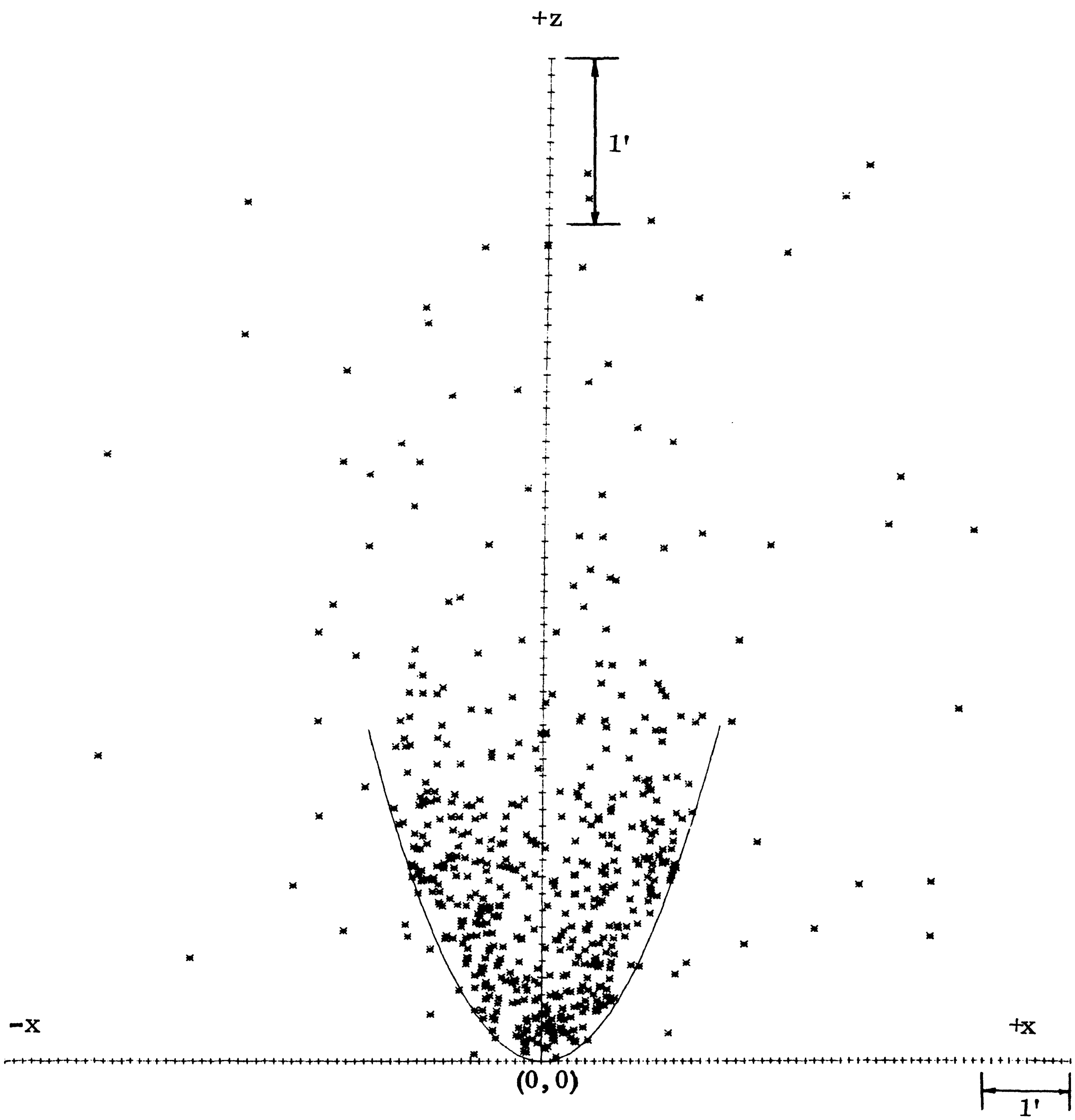


Fig. 6. Y Plane = 1.0 for Infinite Parabola and Neutrons Impinging at $x = -1.5, -.75, 0.0, .75, 1.5$.

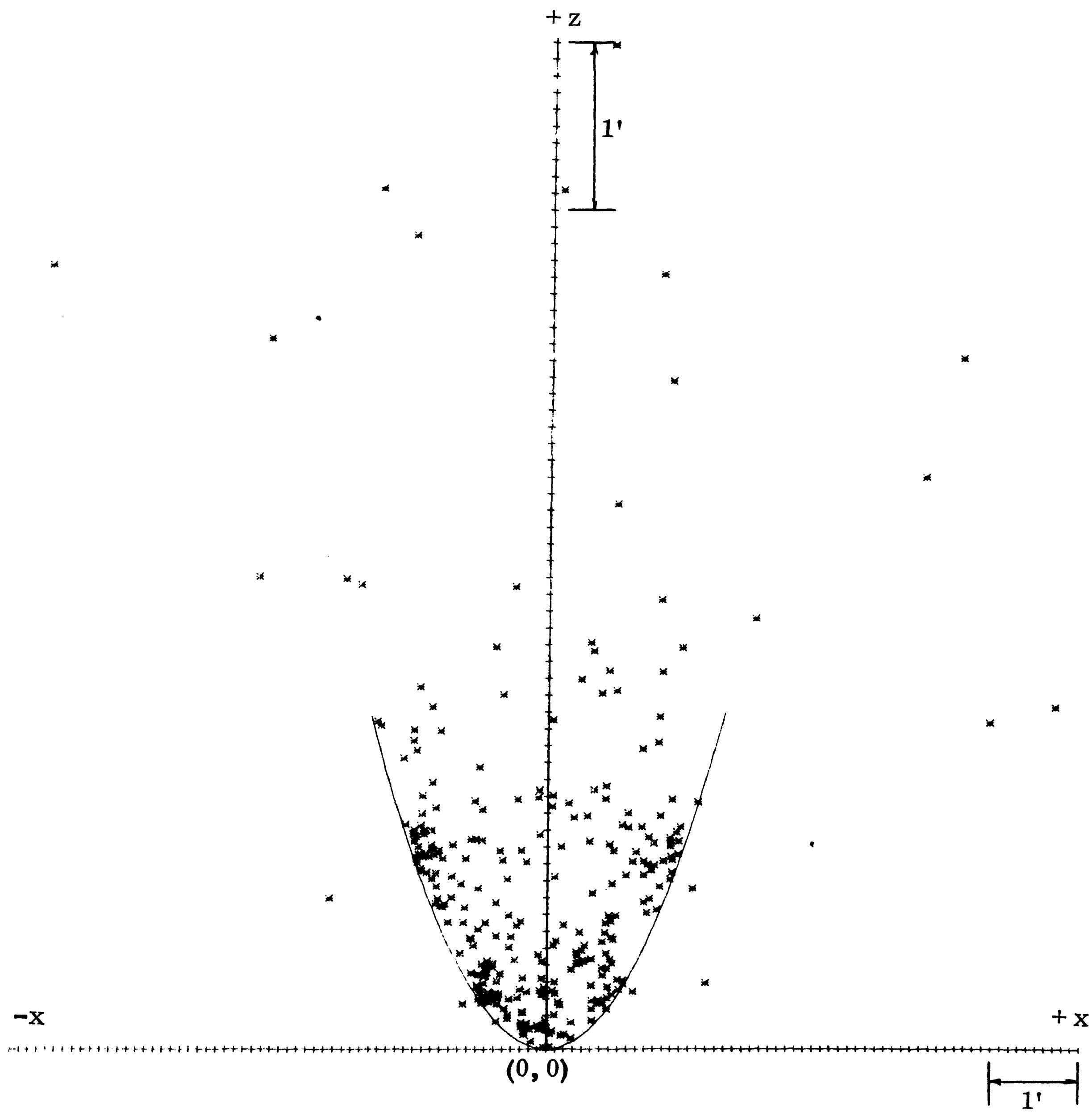


Fig. 7. Y Plane = 1.5 for Infinite Parabola and Neutrons Impinging at
x = -1.5, -.75, 0.0, .75, 1.5.

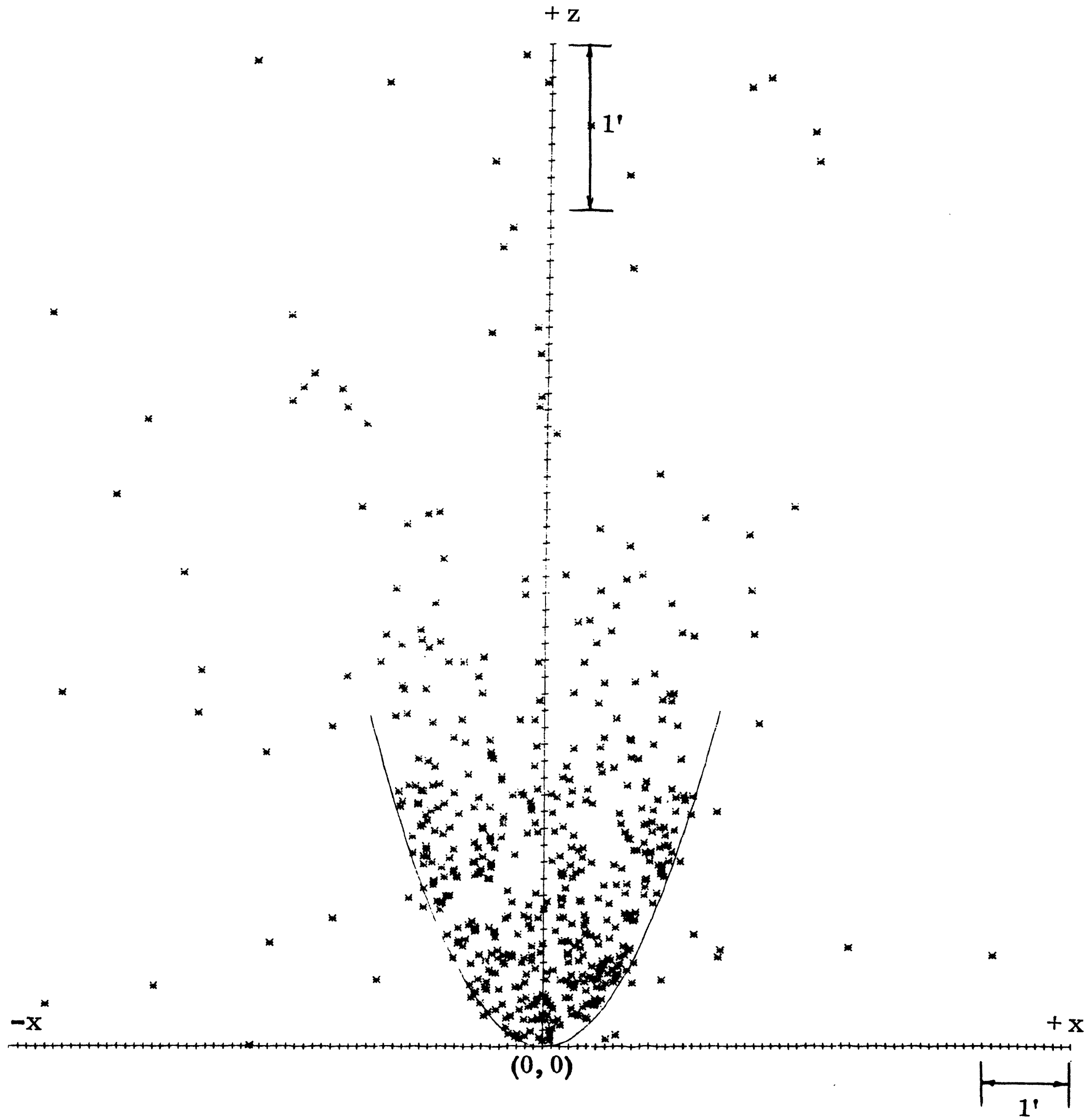


Fig. 8. Y Plane = 2.0 for Infinite Parabola and Neutrons Impinging at x = −1.5, −.75, 0.0, .75, 1.5.

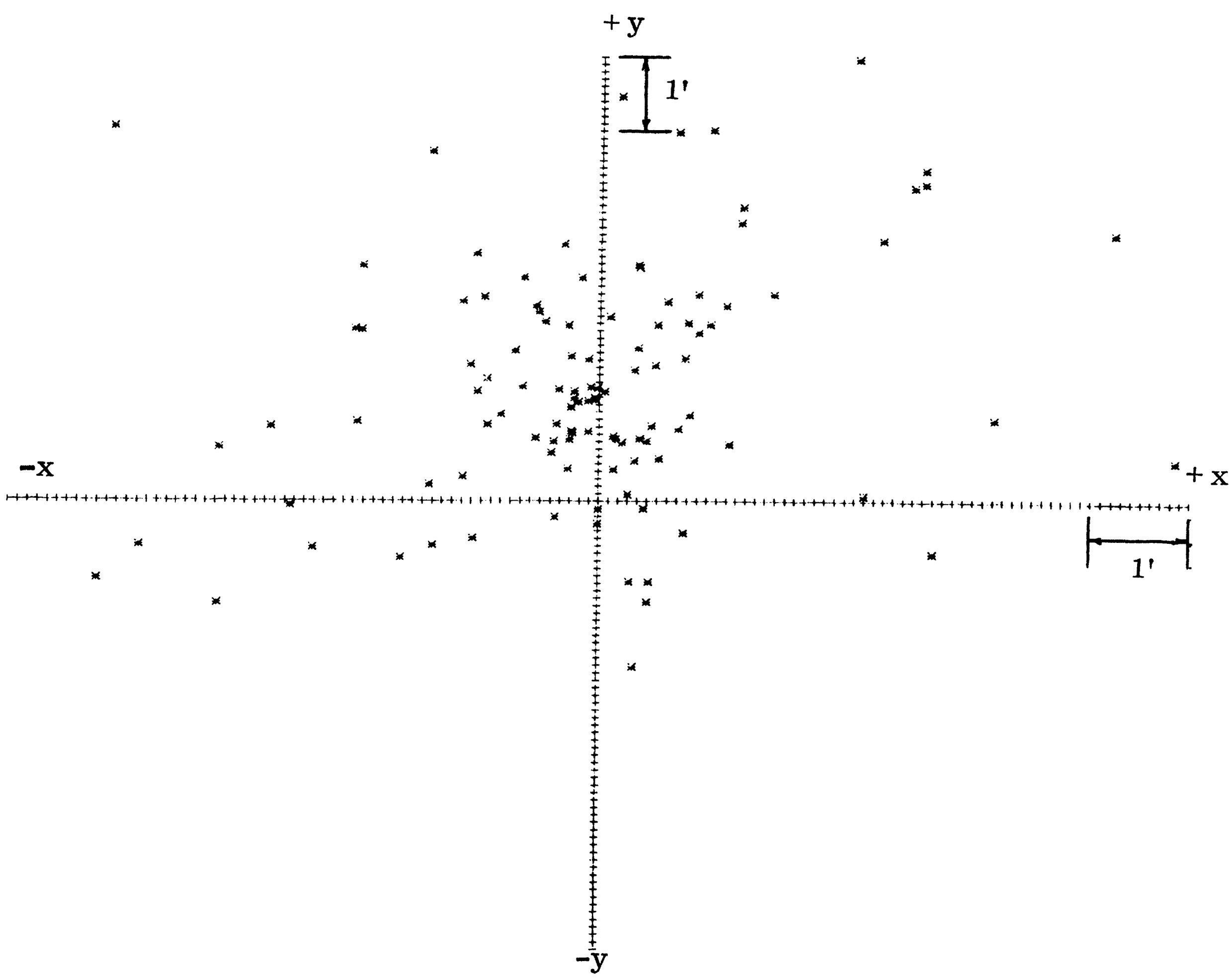


Fig. 9. Z Plane = 0.0 for Infinite Parabola and Neutrons Impinging at x = -1.5, -.75, 0.0, .75, 1.5.

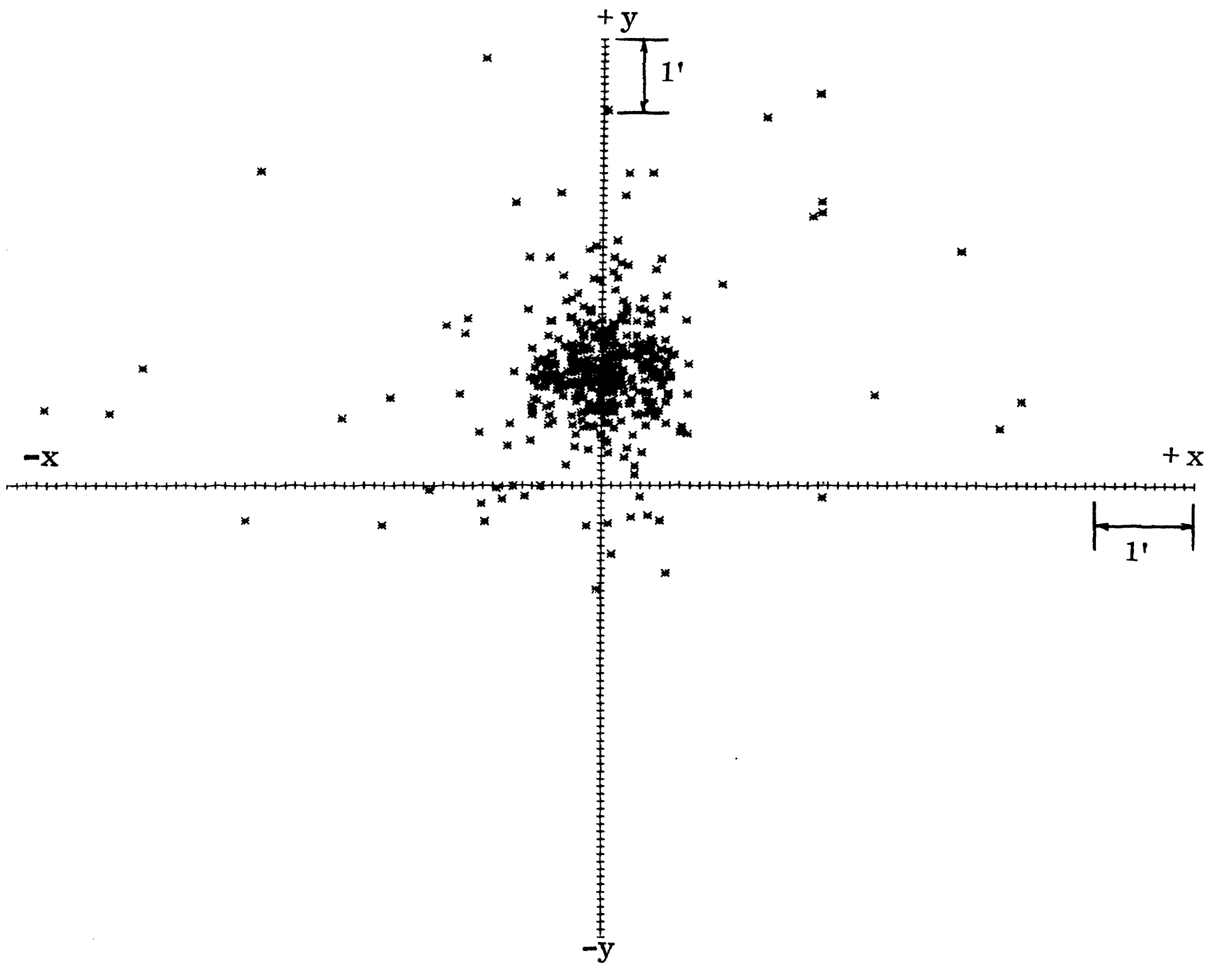


Fig. 10. Z Plane = 0.25 for Infinite Parabola and Neutrons Impinging at
x = -1.5, -.75, 0.0, .75, 1.5.

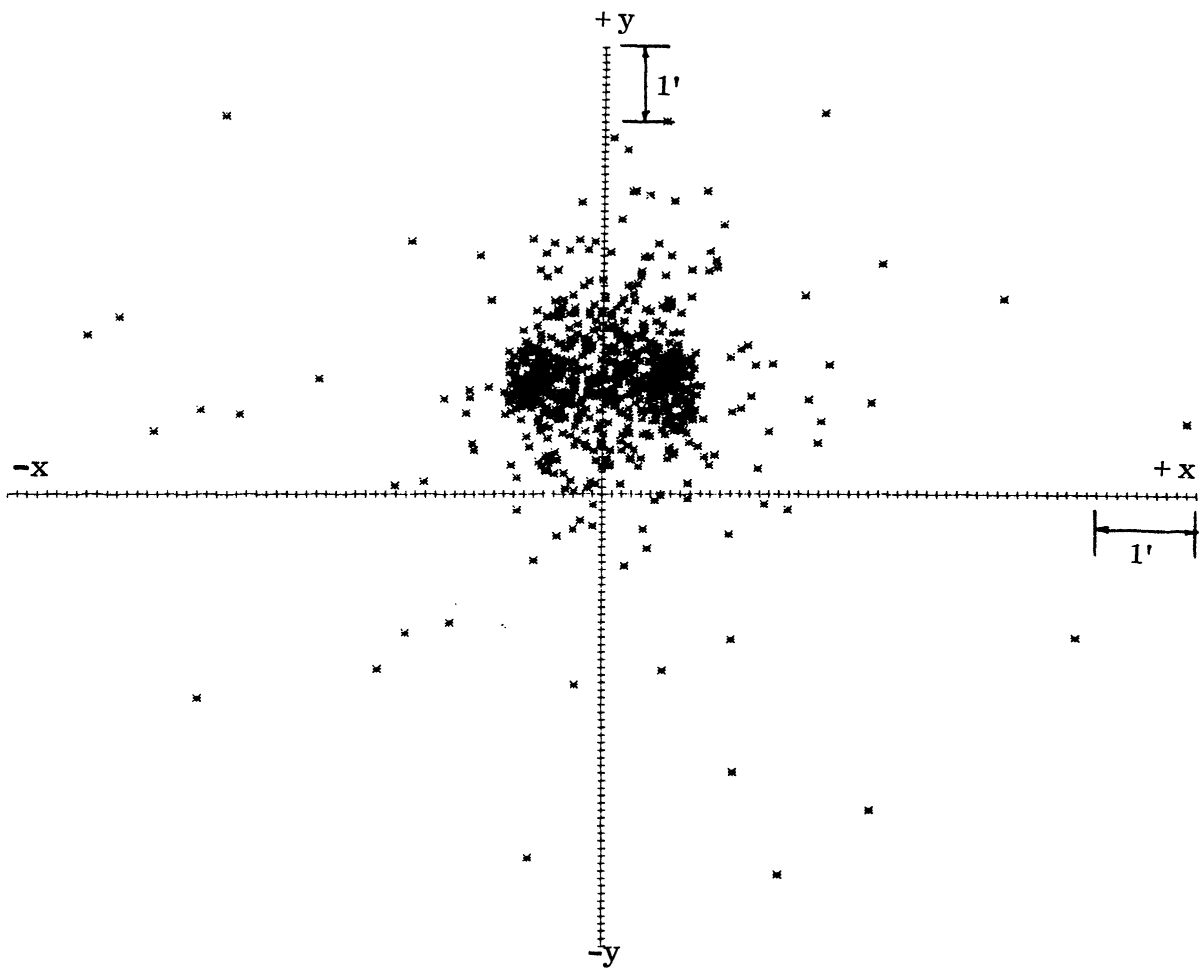


Fig. 11. Z Plane = 0.50 for Infinite Parabola and Neutrons Impinging at x = -1.5, -.75, 0.0, .75, 1.5.

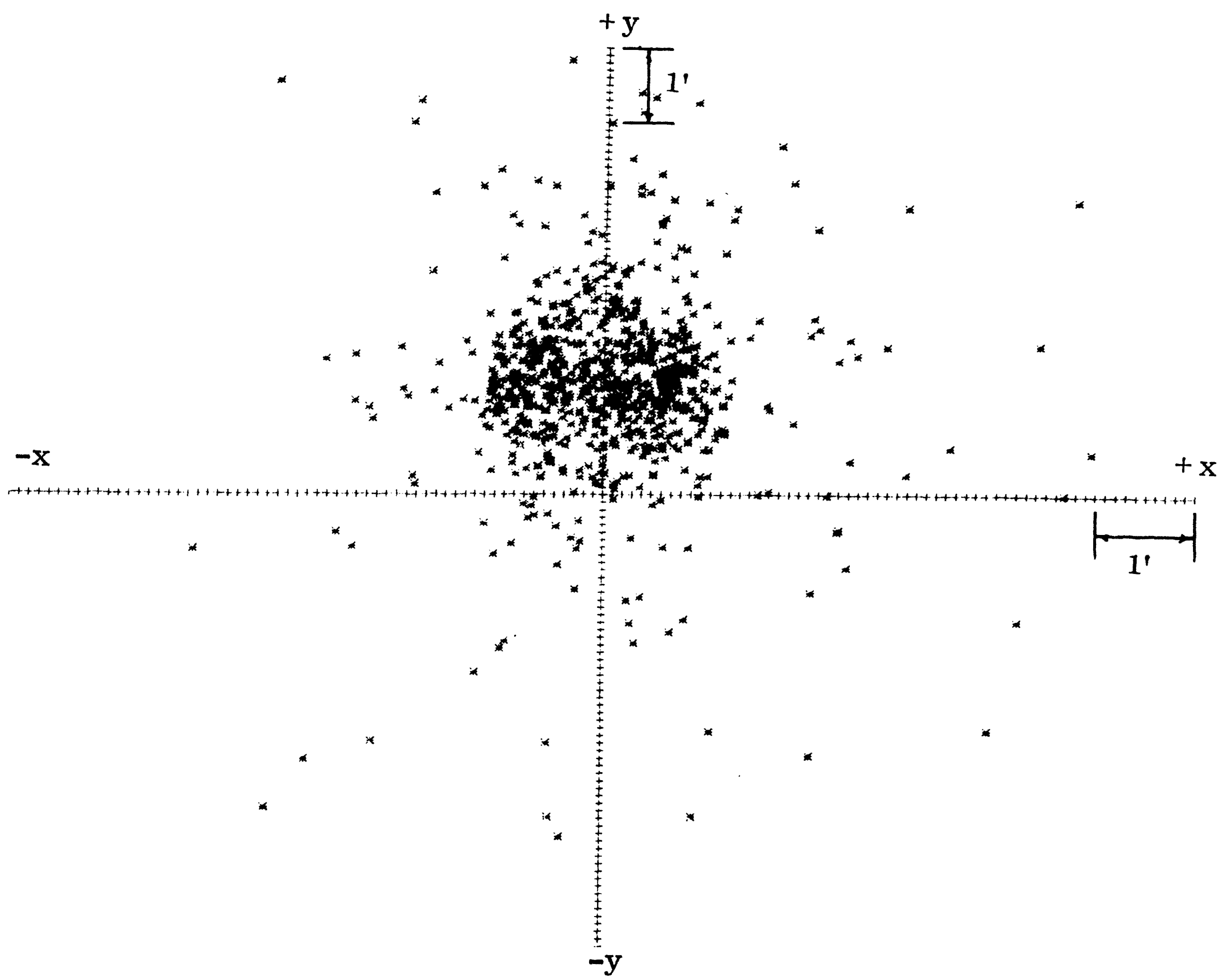


Fig. 12. Z Plane = 0.75 for Infinite Parabola and Neutrons Impinging at x = -1.5, -.75, 0.0, .75, 1.5.

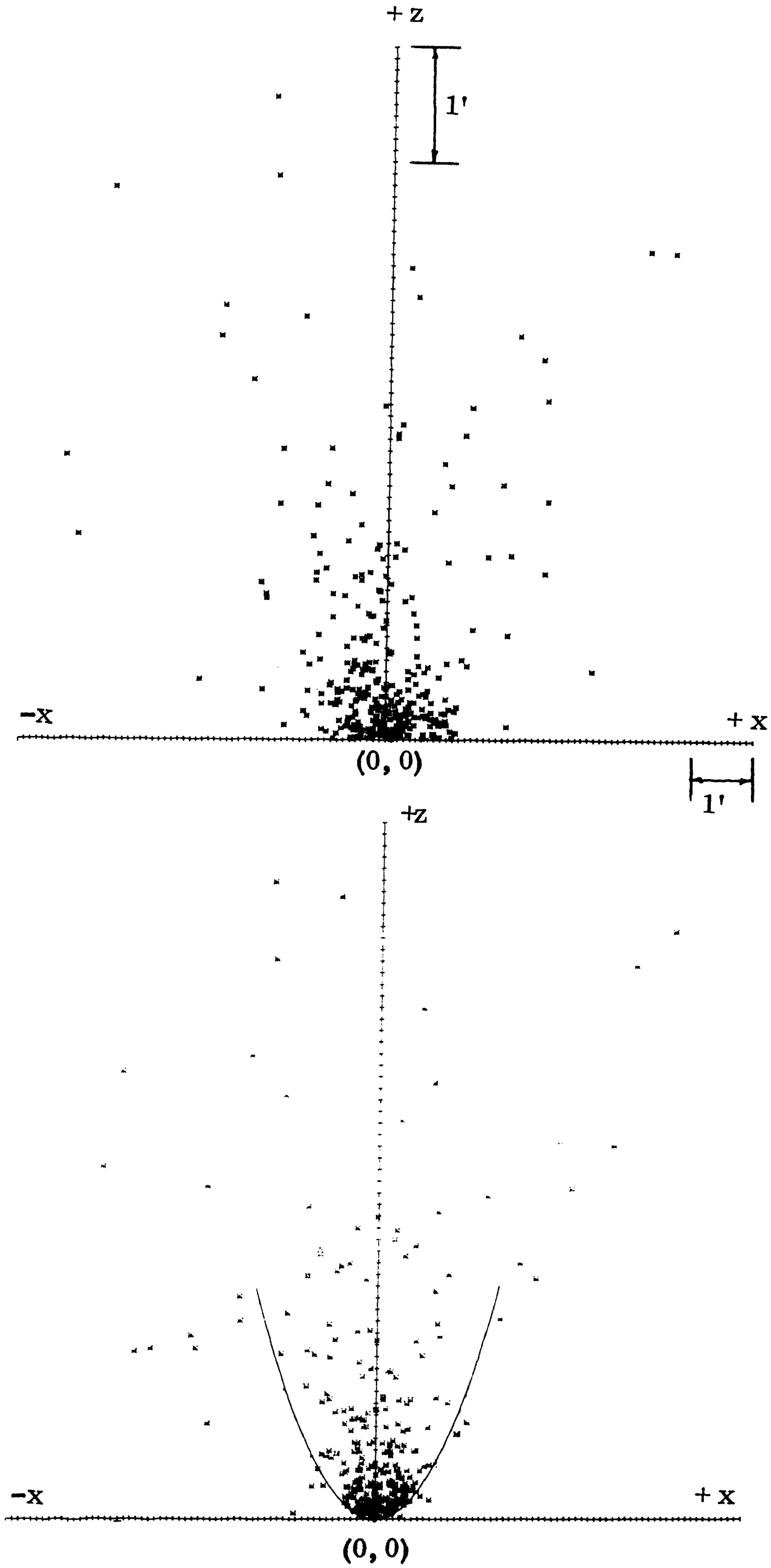


Fig. 13. Comparison of Y Plane = 1.5 for Infinite Slab (top) and Infinite Parabola (bottom) with Neutrons Impinging at x = -0.5, -0.25, 0.0, 0.25, 0.5.

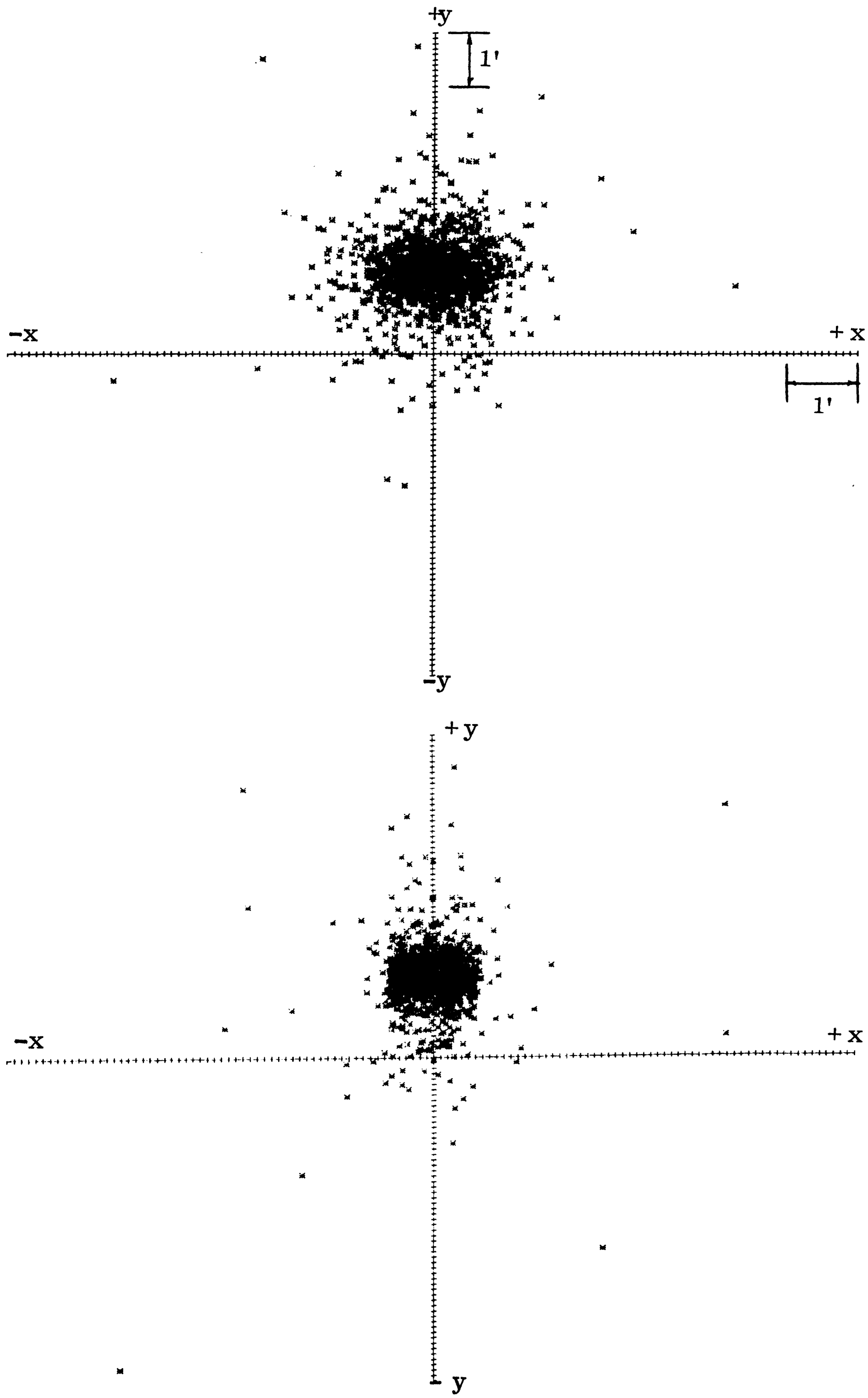


Fig. 14. Comparison of Z Plane = 0.25 for Infinite Slab (top) and Infinite Parabola (bottom) with Neutrons Impinging at x = -0.5, -0.25, 0.0, 0.25, 0.5.

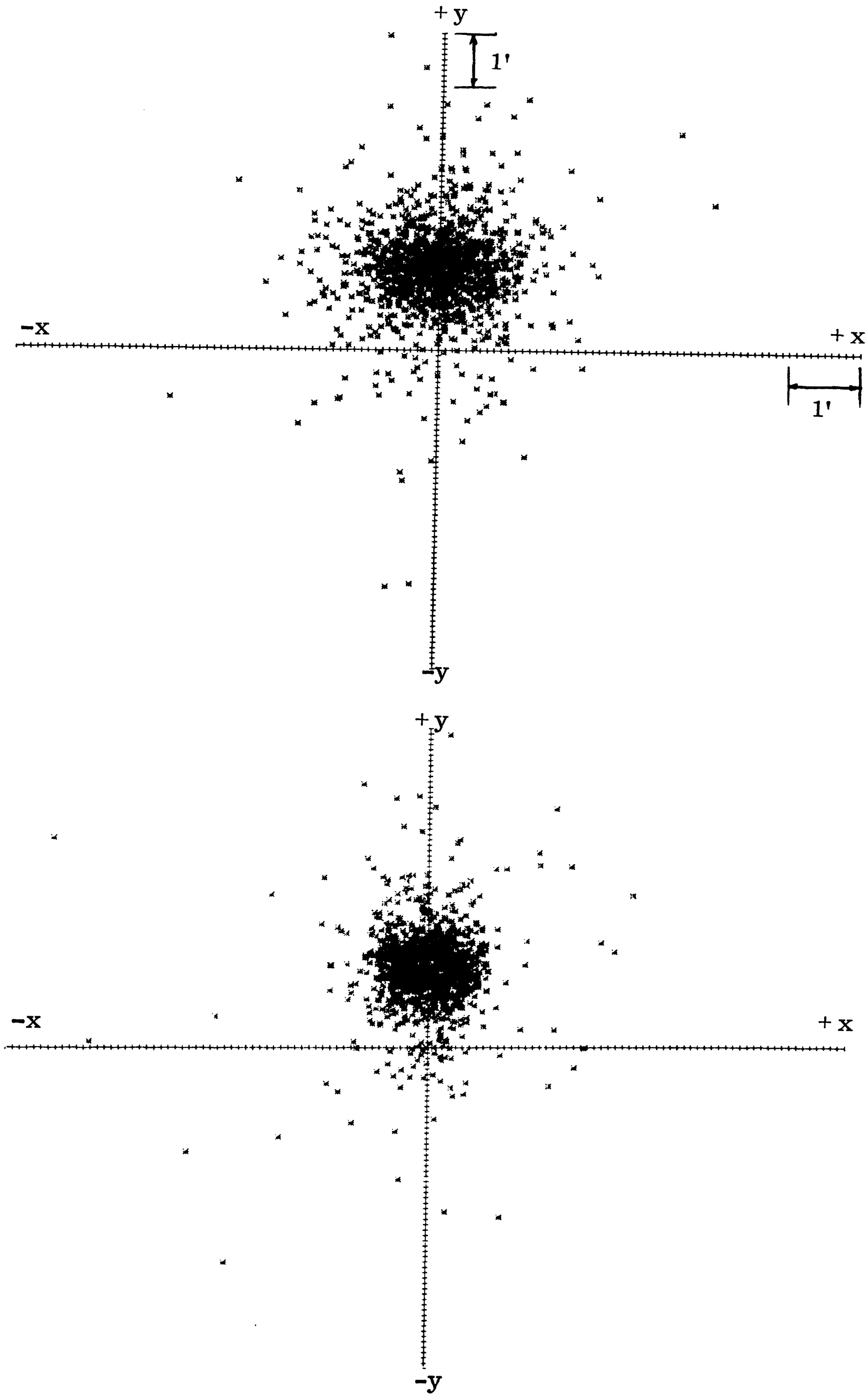
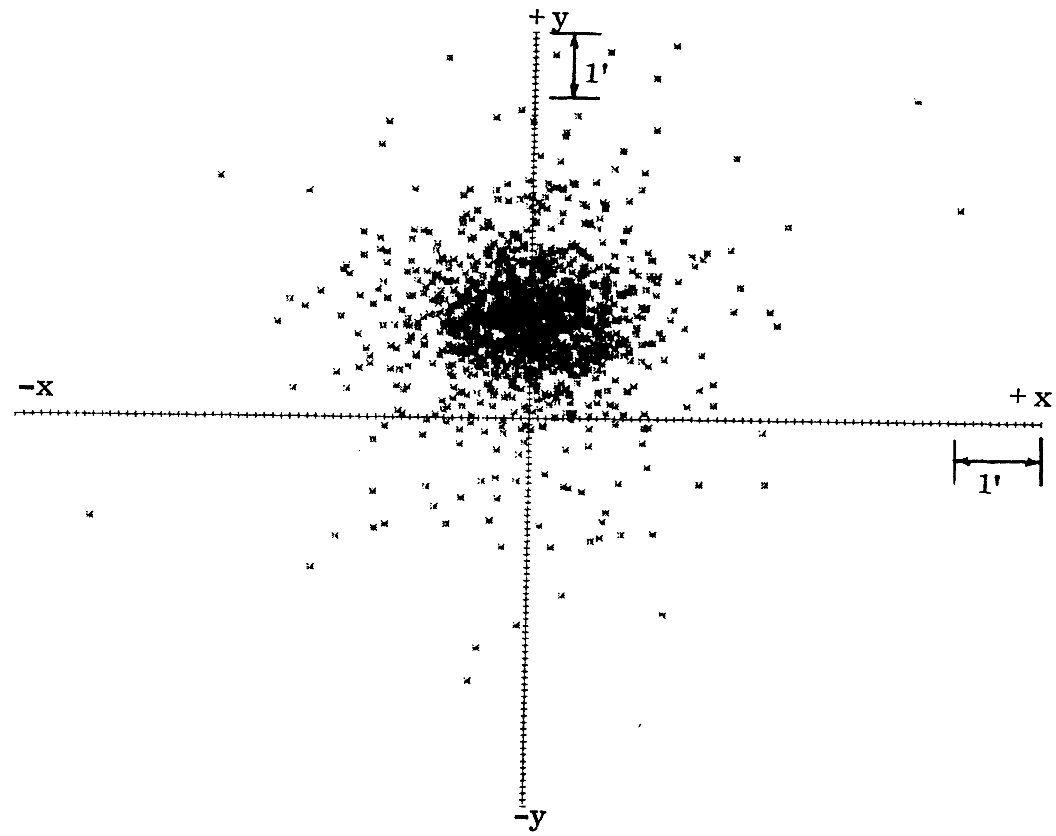


Fig. 15. Comparison of Z Plane = 0.375 for Infinite Slab (top) and Infinite Parabola (bottom) with Neutrons Impinging at x = -0.5, -0.25, 0.0, 0.25, 0.5.



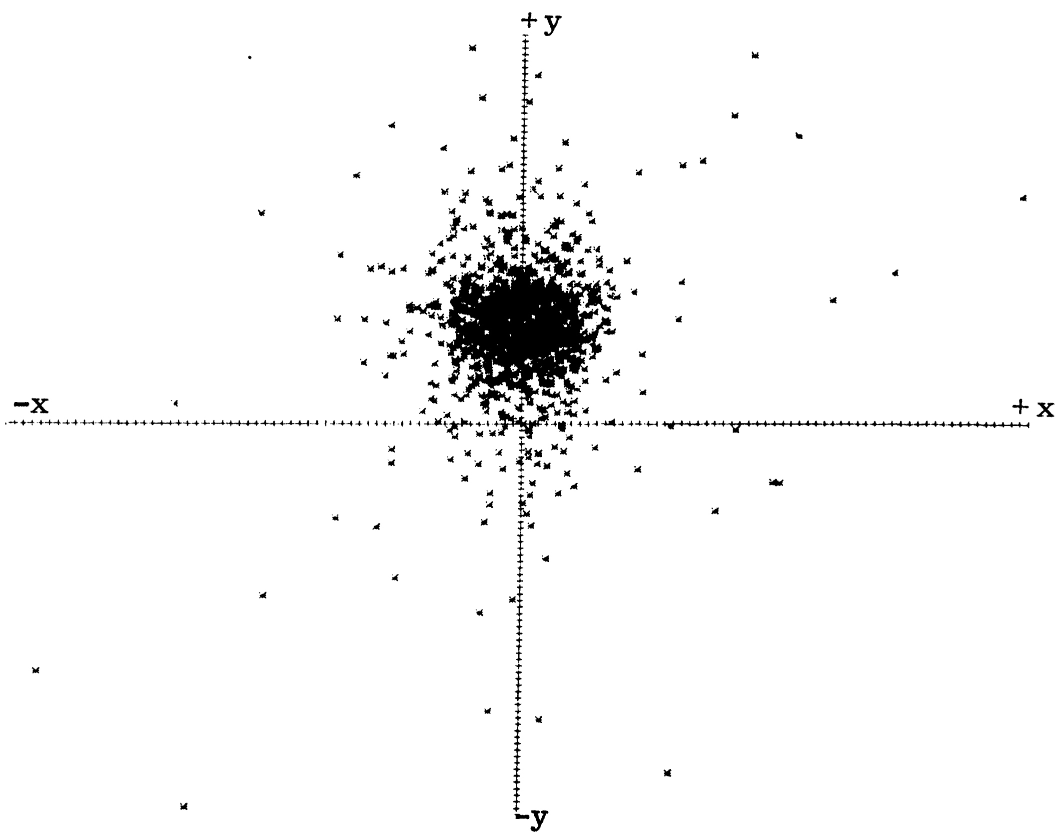


Fig. 16. Comparison of Z Plane = 0.500 for Infinite Slab (top) and Infinite Parabola (bottom) with Neutrons Impinging at x = -0.5, -0.25, 0.0, 0.25, 0.5.

V. DISCUSSION OF RESULTS

The first plane that showed an interesting result is in Fig. 7. This figure is for the Y plane 1.5, which is the position of the incoming neutrons. This figure shown that the majority of the neutrons are re-emitted very close to their input point and thus intercept the Y plane = 1.5 before traveling very far. This explains the clustering of the penetrations near the input point of the neutrons.

The next planes of interest are the Z planes of Figs. 10 and 11. In Fig. 10 (Z plant = 0.25), there is one cluster of neutrons; but in Fig. 11 there are two definite clusters of neutrons. This is due to the fact that discrete neutron input points were used. The reflected beams are more intense in the direction perpendicular to the surface at the point of incidence. At z = .25 the beams coincide and give one cluster of neutrons. At z = .5 the two beams travel in two different directions and produce two clusters.

A parabola with vertex at (h, k) with axis parallel to Z axis, and with the directed distance from the vertex to the focus given by p is the graph of

$$(x - k)^2 = 4p\ (Z - h) \tag{16}$$

(h, k) in this case is (0, 0), so we have

$$x^2 = 4\ pZ \tag{17}$$

and

$$z^2 = 2Z \qquad \text{(computer program equation)} \tag{18}$$

$$p = .5$$

as stated above.

It is a well known fact from physics that a plane wave can be focused at the focal point of a parabola. It appears neutrons also show a "focusing" effect but do not behave as a plane wave, i.e., they do not focus at the same point.

Figs. 14, 15, and 16 show a very interesting result. These figures are the results of neutrons impinging from $x = -.50$ to $x = .50$. They show that even in the section of the parabola where very little shape effect is encountered by the incoming neutron, there is a definite tightening of the neutron reflection pattern from a parabolic surface over that from an infinite slab.

Figures 17-20 show the angular distribution of neutrons reflected from a parabola and an infinite slab as obtained from the Monte Carlo calculations. Fermi's Equation (Fig. 1) is also shown for comparison. The agreement with Fermi's Equation is very good especially for the infinite slab. The results for the parabola show similarity. It should be pointed out that Fermi's Equation is valid for a plane.

TABLE I.

DATA USED IN FIGURE 17

	θ^{o}	Raw Count	Normalized Count
Tan^{-1} 0.0	0.0	318	1.000
Tan^{-1} 1.2	50.2	177	.557
Tan^{-1} 2.4	67.4	83	.261
Tan^{-1} 3.6	74.5	43	.135
Tan^{-1} 4.8	78.2	13	.041
Tan^{-1} 6.0	80.5	8	.025
Tan^{-1} 7.2	82.1	6	.019
Tan^{-1} 8.4	83.2	5	.016
Tan^{-1} 9.6	84.1	2	.006

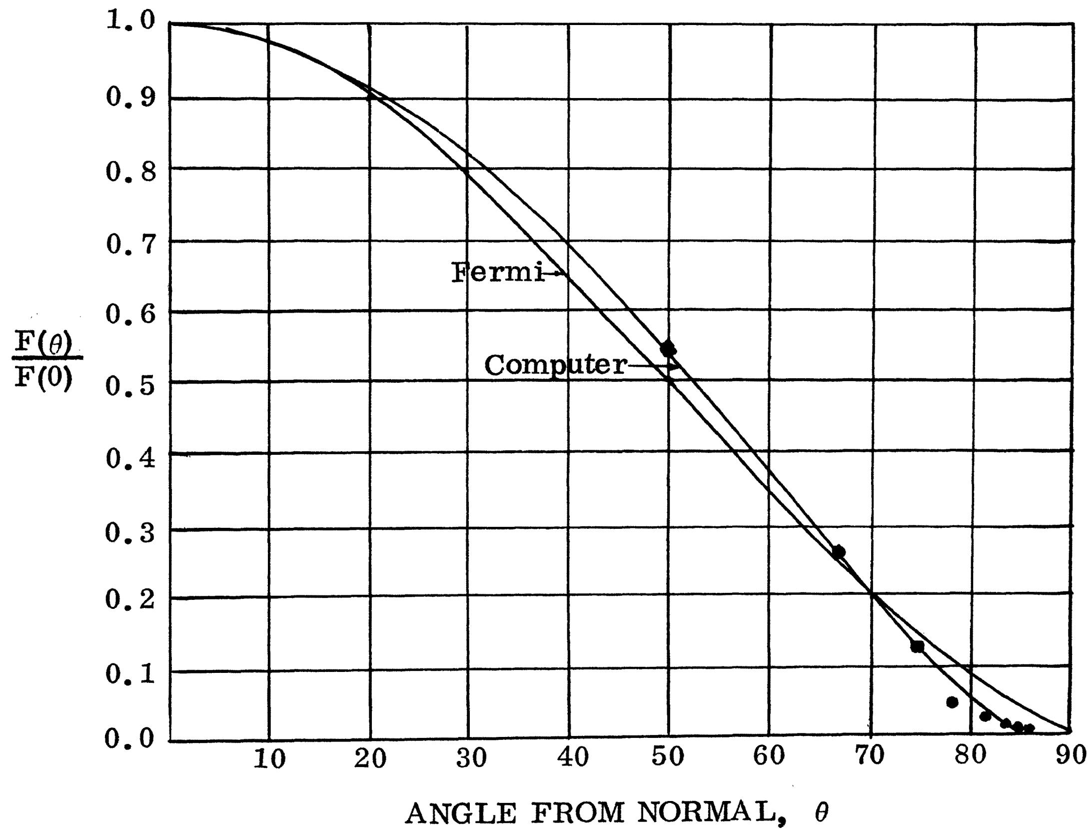


Fig. 17. Comparison of Computer Output to Fermi's Equation for the Infinite Slab and for Z-Plane 0.250.

TABLE II

DATA USED IN FIGURE 18

	θ^o	Raw Count	Normalized Count
Tan^{-1} 0.0	0.0	267	1.000
Tan^{-1} 0.8	38.7	188	.703
Tan^{-1} 1.6	58.0	91	.341
Tan^{-1} 2.4	67.4	50	.187
Tan^{-1} 3.2	72.6	27	.101
Tan^{-1} 4.0	77.0	20	.075
Tan^{-1} 4.8	78.2	9	.034
Tan^{-1} 5.6	79.9	5	.019
Tan^{-1} 6.4	81.1	3	.011
Tan^{-1} 7.2	82.1	3	.011

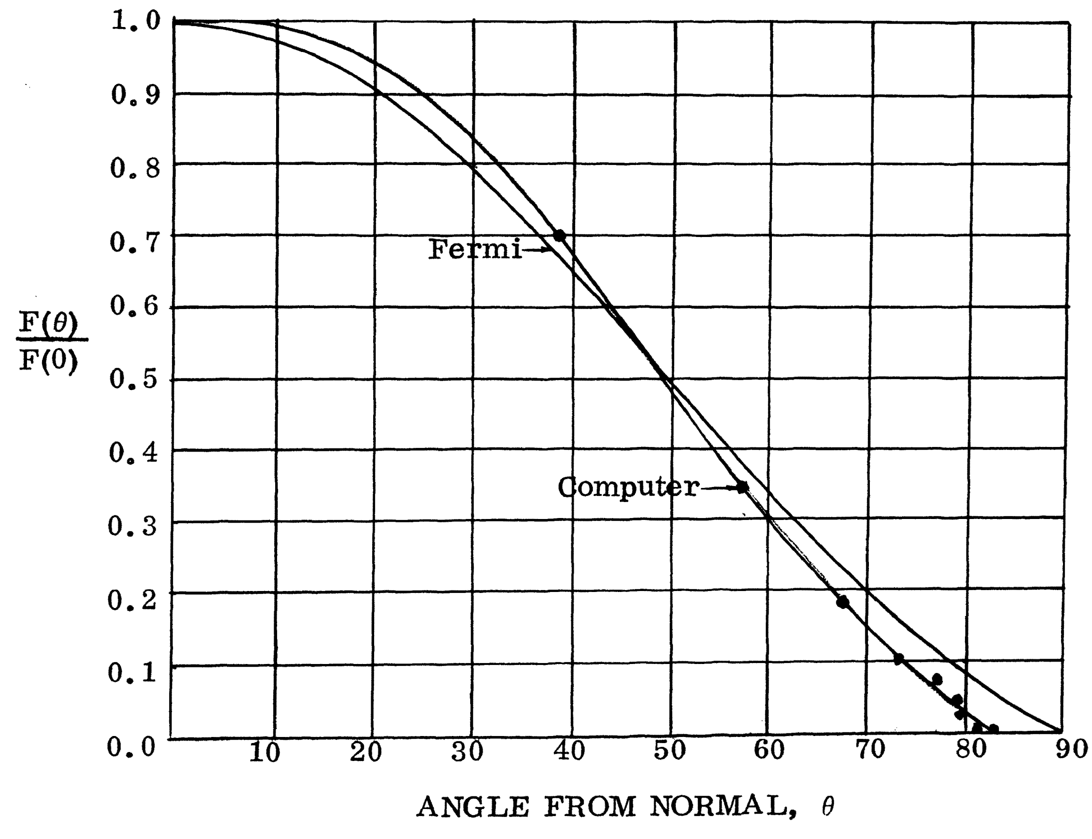


Fig. 18. Comparison of Computer Output to Fermi's Equation for the Infinite Slab and for Z-Plane 0.375.

TABLE III

DATA USED IN FIGURE 19

	θ^{o}	Raw Count	Normalized Count
Tan^{-1} 0.0	0.0	225	1.000
Tan^{-1} 0.6	31.0	178	.780
Tan^{-1} 1.2	50.2	102	.452
Tan^{-1} 1.8	61.0	64	.284
Tan^{-1} 2.4	67.4	28	.124
Tan^{-1} 3.0	71.6	20	.089
Tan^{-1} 3.6	74.5	23	.102
Tan^{-1} 4.0	76.6	10	.044
Tan^{-1} 4.6	78.2	4	.018
Tan^{-1} 5.4	79.5	4	.018
Tan^{-1} 6.0	80.5	5	.022
Tan^{-1} 6.6	81.5	0	.000

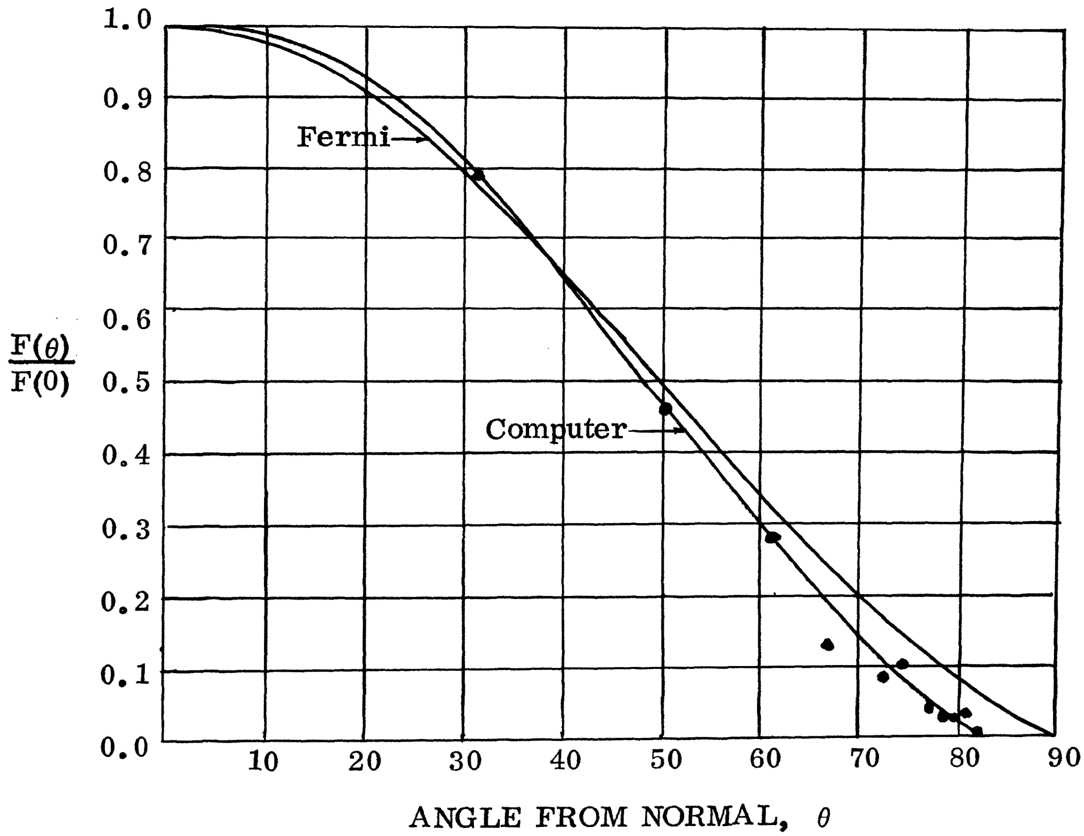


Fig. 19. Comparison of Computer Output to Fermi's Equation for the Infinite Slab and for Z-Plane 0.500.

TABLE IV

DATA USED IN FIGURE 20

	θ^{o}	Raw Count	Normalized Count
Tan^{-1} 0.0	0.0	195	1.000
Tan^{-1} 0.6	31.0	169	.867
Tan^{-1} 1.2	50.2	84	.430
Tan^{-1} 1.8	61.0	44	.226
Tan^{-1} 2.4	67.4	24	.123
Tan^{-1} 3.0	71.6	21	.108
Tan^{-1} 3.6	74.5	10	.051
Tan^{-1} 4.0	76.6	6	.031
Tan^{-1} 4.6	78.2	5	.026
Tan^{-1} 5.4	79.5	4	.021
Tan^{-1} 6.0	80.5	2	.011
Tan^{-1} 6.6	81.5	0	.000

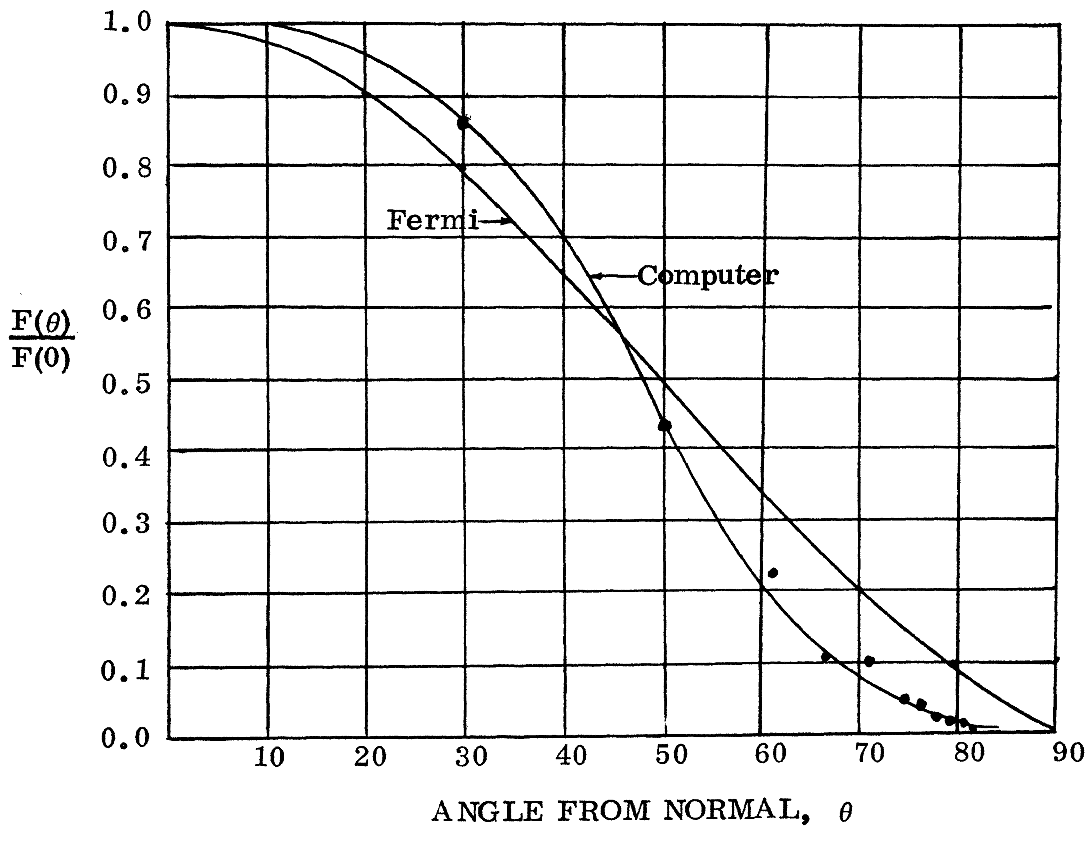


Fig. 20. Comparison of Computer Output to Fermi's Equation for the Infinite Parabola and for Z-Plane 0.5.

VI. RELIABILITY OF THE RESULTS

The results of this thesis were obtained with the use of a Monte Carlo calculation. As explained earlier, the Monte Carlo Method employs probabilistic concepts; therefore, any results obtained with it will be subjected to statistical uncertainties. It is not easy to calculate statistical uncertainties of Monte Carlo results. However, the uncertainties are in general due to the finite number of particle histories studied. For example, if reflection is studied and the calculation shows that N particles were reflected, the uncertainty of this number is close to $1/\sqrt{N}$.

The work presented here did not deal with calculations of specific quantities. Instead, patterns of reflected neutrons were studied. In all programs 1,500 neutron histories were followed. Out of these about 1,200 were reflected. The validity of the Monte Carlo calculation was checked when the author wrote a program and calculated a neutron albedo. The albedo values obtained were, within statistical limits, identical to those calculated using the diffusion equation.

The agreement of the Monte Carlo calculation results with the Fermi equation (Figs. 17-20) is another indication that the computer program, as written by the author, gives reliable information.

Apart from the fact that a finite number of neutron histories was studied, the following two approximations introduced some uncertainty into the results.

First, the assumption of isotropic scattering. This should not be significant (see p. 7 and ref. 9). However, an improved calculation should include crystalline effects of the medium and consequently anisotropic scattering.

Second, the assumption that the neutron was absorbed if still in the medium after 150 collisions. The error due to this assumption is less than the error due to the statistical nature of the method itself, because less than 10% of the neutrons survived 150 collisions.

VII. CONCLUSIONS AND RECOMMENDATIONS

This work has demonstrated that there is a definite difference between the pattern of neutrons reflected from an infinite slab and that of neutrons reflected from a parabola. The reflected neutron beam in the case of the parabola is more concentrated (more "focused"). Additional study is recommended in the following three areas:

1. An experiment,
2. Paraboloid,
3. Improved calculation including crystalline effects.

An experimental duplication would not be too dificult to set up. The author's conception of an experiment using a water medium would be a parabolic tank made out of polyethylene and filled with water. Neutrons provided by the reactor beam port would impinge upon the parabolic tank, enter it, and diffuse. Reflected neutrons could be detected at selected positions.

Since there was ordering of neutrons using a parabolic reflector, it would appear that additional ordering would be produced by using a paraboloid reflector.

VIII. APPENDICES

APPENDIX A

COORDINATE SYSTEM USED IN COMPUTER PROGRAM

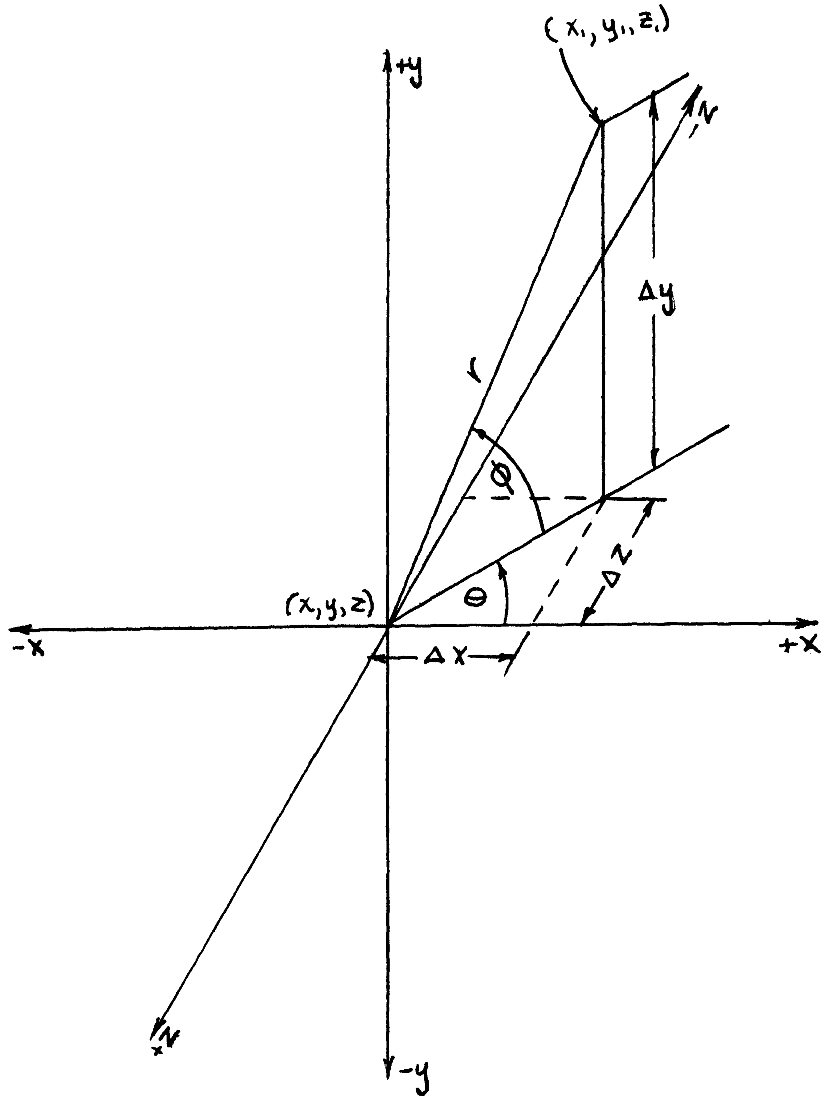


Fig. 21. Coordinate System Used in Computer Program

Comments on Coordinate System:

Neutrons for the infinite slab and for the infinite parabola were sent in parallel to the Z-axis and traveling in the negative Z direction. The coordinate system used is spherical coordinates with each position always calculated and stored in cartesian coordinates.

$$\Delta x = r \cos \varphi \cos \theta$$

$$\Delta z = r \cos \varphi \sin \theta$$

$$\Delta y = r \sin \theta$$

$$x_1 = x + \Delta x$$

$$y_1 = y + \Delta y$$

$$z_1 = z + \Delta z$$

The neutron has a traveling coordinate system with its final position always calculated with respect to its previous coordinates as indicated in the above equations.

APPENDIX B

COMPUTER PROGRAM

A. Computer Program

```
C    THE FOLLOWING PROGRAM IS A MONTE CARLO TECHNIQUE FOR THE STUDY OF
C      THE SHAPE EFFECT ON THE SCATTER OF NEUTRONS IMPINGING ON A SURFACE
C      THIS RUN IS FOR NEUTRONS IMPINGING ON AN INFINITE PARABOLIC REFLECTOR
       DIMENSION XP1(1500),ZP1(1500),YP1(1500)
       DIMENSION XP2(1500),ZP2(1500),YP2(1500)
       DIMENSION X1(1500),Y1(1500),Z1(1500),T(1500),P(1500)
       DIMENSION XPC(300),YPC(300)
       K22=0
       N=0
       REAL LAMDA
       REAL NN2
       REAL NN1
       SIGS=(3.85E-01)*30.43
       SIGA=(3.2E-04)*30.43
       PA=SIGZ/(SIGA+SIGS)
       PS=SIGS/(SIGA+SIGS)
       CALL PENPOS('RACKLEY JAY',11,1)
       DO 3 J=1,5
       DO 30 M=1,300
       X=-0.5+(J-1)*.25
       Y=1.5
       Z=(X**2)/2.
       THETA=3.14/2.
       PHI=0.0
       L=1
       S=RAND(1)
       LAMDA=-(1./(SIGA+SIGS))*ALOG(S)
       Z=Z-LAMDA
    11 S=RAND(1)
       IF(S.LT.PA)GO TO 27
       S=RAND(1)
       LAMDA=-(1./(SIGA+SIGS))*ALOG(S)
       IF(LAMDA.EQ.0)GO TO 5
       S=RAND(1)
```

```
      THETA=6.28*S
      S=RAND(1)
      PHI=((3.14)/2.)*S
      S=RAND(1)
      IF(S.LE. .5)GO TO 15
      GO TO 88
   15 PHI=-PHI
   88 DELZ=LAMDA*COS(PHI)*SIN(THETA)
      DELX=LAMDA*COS(PHI)*COS(THETA)
      DELY=LAMDA*SIN(PHI)
      Z=Z-DELZ
      X=X+DELX
      Y=Y+DELY
      IF(Z.GE.2.0.AND.X.LE.-2.0)GO TO 27
      IF(Z.GE.2.0.AND.X.GE.2.0)GO TO 27
      ZCURV=(X**2)/2.
      IF(Z.GT.ACURV)GO TO 9
      IF(L.GE.150)GO TO 12
      GO TO 5
   27 N=N+1
      GO TO 30
    5 L=L+1
      GO TO 11
   12 WRITE(3,500)L
  500 FORMAT(10X,'NEUTRON WAS NOT ABSORBED OR SCATTERED AFTER',2X,I5,
     12X,'COLLISIONS')
      N=N+1
      GO TO 30
    9 K22=K22+1
      X1(K22)=X
      Y1(K22)=Y
      Z1(K22)=Z
      T(K22)=THETA
      P(K22)=PHI
   30 CONTINUE
    3 CONTINUE
```

```
      WRITE(3,111)K22
  111 FORMAT(5X,'K22=',1X,I5)
      DO 26 I=1,3
      LM1=0
      LL=0
      LL1=0
      LL2-0
      LL3=0
      LL4=0
      LL5=0
      LL6=0
      LL7=0
      LL8=0
      NN1=0.0
      NN2=0.0
      YP=1.5+(I-1)*.5
      K2K=1
   14 X=X1(K2K)
      Y=Y1(K2K)
      Z=Z1(K2K)
      THETA=T(K2K)
      PHI=P(K2K)
      IF(YP.GE.Y.AND.PHI.LE.0.0)GO TO 25
      IF(YP.LE.Y.AND.PHI.GE.0.0)GO TO 25
      D=(YP-Y)/SIN(PHI)
      X=X+D*COS(PHI)*COS(THETA)
      Y=Y+D*SIN(PHI)
      Z=Z-D*COS(PHI)*SIN(THETA)
      IF(Z.LT.0)GO TO 52
      IF(Z.GE.0..AND.Z.LE..5)GO TO 5000
      IF(Z.GT..5.AND.Z.LE.1.0)GO TO 5001
      IF(Z.GT.1.0.AND.Z.LE.2.0)GO TO 5002
      IF(Z.GT.2.0.AND.Z.LE.3.0)GO TO 5003
      IF(Z.GT.3.0.AND.Z.LE.4.0)GO TO 5004
      IF(Z.GT.4.0.AND.Z.LE.5.0)GO TO 5005
      IF(Z.GT.5.0.AND.Z.LE.10.0)GO TO 5006
```

```
      IF(Z.GT.10.0.AND.Z.LE.15.0)GO TO 5007
      IF(Z.GT.15.0)GO TO 5008
 5000 LL=LL+1
      GO TO 52
 5001 LL1=LL1+1
      GO TO 52
 5002 LL2=LL2+1
      GO TO 52
 5003 LL3=LL3+1
      GO TO 52
 5004 LL4=LL4+1
      GO TO 52
 5005 LL5=LL5+1
      GO TO 52
 5006 LL6=LL6+1
      GO TO 52
 5007 LL7=LL7+1
      GO TO 52
 5008 LL8=LL8+1
   52 NN2=NN2+1
      IF(Z.LT.0.0.OR.Z.GE.6.)GO TO 1000
      IF(ABS(X).GE.6.)GO TO 1000
      LM1=LM1+1
      XP1(LM1)=X
      ZP1(LM1)=Z
      GO TO 25
 1000 NN1=NN1+1
   25 IF(K2K.GE.K22)GO TO 86
      K2K=K2K+1
      GO TO 14
   86 WRITE(3,921)YP
  921 FORMAT(5X,'THE FOLLOWING PLOT IS FOR Y PLANE=',2X,F11.4)
      WRITE(3,2124)LM1
 2124 FORMAT(5X,'LM1=',1X,I5)
      IF(LM1.LT.1.0)GO TO 444
      WRITE(3,5555)LL,LL1,LL2,LL3,LL4,LL5,LL6,LL7,LL8
```

```
5555 FORMAT(5X,9I5)
     FRACT=NN1/NN2
     WRITE(3,1001)FRACT
1001 FORMAT(5X,'FRACT THAT INTERCEPTED PLANE OUT OF BOUNDS =',1X,F11.4)
     CALL PPLT(XP1,ZP1,LM1)
     CALL NEWPLT(5.00,0.00,11.)
     CALL ORIGIN(0.0,0.0)
     CALL XSCALE(-6.,6.,10.)
     CALL YSCALE(0.,6.,10.)
     CALL XAXIS(.1)
     CALL YAXIS(.1)
     CALL XYPLT(XP1,ZP1,LM1,2,11)
     DO 51 NN=1,101
     XPC(NN)=-2.0+(NN-1)*.04
  51 YPC(NN)=XPC(NN)**2/2.
     CALL XYPLT(XPC,YPC,101,1,-11)
     CALL ENDPLT
     GO TO 26
 444 WRITE(3,134)
 134 FORMAT(5X,'INSUFFICIENT NUMBER OF POINTS')
  26 CONTINUE
     DO 7 II=1,5
     ZP=.25+(II-1)*.125
     LLL=0
     LLL1=0
     LLL2=0
     LLL3=0
     LLL4=0
     LLL5=0
     LLL6=0
     LLL7=0
     LLL8=0
     LLL9=0
     LLL10=0
     LLL11=0
     LLL12=0
```

```
      LLL13=0
      LLL14=0
      LLL15=0
      LLL16=0
      LLL17=0
      LLL18=0
      LLL19=0
      LLL20=0
      LLL21=0
      LM2=0
      KK=1
   43 X=X1(KK)
      Y=Y1(KK)
      Z=Z1(KK)
      THETA=T(KK)
      PHI=P(KK)
      IF(ZP.LE.Z.AND.THETA.GE.3.14)GO TO 45
      IF(ZP.GE.Z.AND.THETA.LE.3.14)GO TO 45
      DZP=(Z-ZP)/(COS(PHI)*SIN(THETA))
      Y=Y+DZP*SIN(PHI)
      X=X+DZP*COS(PHI)*COS(THETA)
      Z=Z-DZP*COS(PHI)*SIN(THETA)
      IF(Y.LT.-1.8.OR.Y.GT.4.8)GO TO 7000
      IF(Y.GE.-1.8.AND.Y.LT.-1.5)GO TO 6000
      IF(Y.GE.-1.5.AND.Y.LT.-1.2)GO TO 6001
      IF(Y.GE.-1.2.AND.Y.LT.-0.9)GO TO 6002
      IF(Y.GE.-0.9.AND.Y.LT.-0.6)GO TO 6003
      IF(Y.GE.-0.6.AND.Y.LT.-0.3)GO TO 6004
      IF(Y.GE.-0.3.AND.Y.LT.0.0)GO TO 6005
      IF(Y.GE.0.0.AND.Y.LT.0.3)GO TO 6006
      IF(Y.GE.0.3.AND.Y.LT.0.6)GO TO 6007
      IF(Y.GE.0.6.AND.Y.LT.0.9)GO TO 6008
      IF(Y.GE.0.9.AND.Y.LT.1.2)GO TO 6009
      IF(Y.GE.1.2.AND.Y.LT.1.5)GO TO 6010
      IF(Y.GE.1.5.AND.Y.LT.1.8)TO TO 6011
      IF(Y.GE.1.8.AND.Y.LT.2.1)GO TO 6012
```

```
      IF(Y.GE.2.1.AND.Y.LT.2.4)GO TO 6013
      IF(Y.GE.2.4.AND.Y.LT.2.7)GO TO 6014
      IF(Y.GE.2.7.AND.Y.LT.3.0)GO TO 6015
      IF(Y.GE.3.0.AND.Y.LT.3.3)GO TO 6016
      IF(Y.GE.3.3.AND.Y.LT.3.6)GO TO 6017
      IF(Y.GE.3.6.AND.Y.LT.3.9)GO TO 6018
      IF(Y.GE.3.9.AND.Y.LT.4.2)GO TO 6019
      IF(Y.GE.4.2.AND.Y.LT.4.5)GO TO 6020
      IF(Y.GE.4.5.AND.Y.LE.4.8)GO TO 6021
      GO TO 7000
 6000 LLL=LLL+1
      GO TO 7000
 6001 LLL1=LLL1+1
      GO TO 7000
 6002 LLL2=LLL2+1
      GO TO 7000
 6003 LLL3=LLL3+1
      GO TO 7000
 6004 LLL4=LLL4+1
      GO TO 7000
 6005 LLL5=LLL5+1
      GO TO 7000
 6006 LLL6=LLL6+1
      GO TO 7000
 6007 LLL7=LLL7+1
      GO TO 7000
 6008 LLL8=LLL8+1
      GO TO 7000
 6009 LLL9=LLL9+1
      GO TO 7000
 6010 LLL10=LLL10+1
      GO TO 7000
 6011 LLL11=LLL11+1
      GO TO 7000
 6012 LLL12=LLL12+1
      GO TO 7000
```

```
6013 LLL13=LLL13+1
     GO TO 7000
6014 LLL14=LLL14+1
     GO TO 7000
6015 LLL15=LLL15+1
     GO TO 7000
6016 LLL16=LLL16+1
     GO TO 7000
6017 LLL17=LLL17+1
     GO TO 7000
6018 LLL18=LLL18+1
     GO TO 7000
6019 LLL19=LLL19+1
     GO TO 7000
6020 LLL20=LLL20+1
     GO TO 7000
6021 LLL21=LLL21+1
7000 IF(ABS(X).GE.6.)GO TO 45
     IF(ABS(Y).GE.6.)GO TO 45
     LM2=LM2+1
     XP2(LM2)=X
     YP2(LM2)=Y
  45 IF(KK.GE.K22)GO TO 87
     KK=KK+1
     GO TO 43
  87 WRITE(3,931)ZP
 931 FORMAT(5X,'THE FOLLOWING PLOT IS FOR Z PLANE=',2X,F11.4)
     WRITE(3,2125)LM2
2125 FORMAT(5X,'LM2=',1X,I5)
     WRITE(3,6030)LLL,LLL1,LLL2,LLL3,LLL4,LLL5,LLL6,LLL7,LLL8,LLL9,
    1LLL10,LLL11,LLL12,LLL13,LLL14,LLL15,LLL16,LLL17,LLL18,LLL19,
    1LLL20,LLL21
6030 FORMAT(3X,20I4)
     IF(LM2.LT.1.0)GO TO 555
     CALL PPLT(XP2,YP2,LM2)
     CALL NEWPLT(5.00,4.25,11.)
```

```
      CALL ORIGIN(0.0,0.0)
      CALL XSCALE(-6.,6.,10.)
      CALL YSCALE(-6.,6.,8.)
      CALL XAXIS(.1)
      CALL YAXIS(.1)
      CALL XYPLT(XP2,YP2,LM2,2,11)
      CALL ENDPLT
      GO TO 7
  555 WRITE(3,134)
      7 CONTINUE
      CALL LSTPLT
      STOP
      END
```

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X. VITA

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